Power-Map Analysis of VLSI Layout by Geometric Techniques

Bhargab B. Bhattacharya
Advanced Computing & Microelectronics Unit
Indian Statistical Institute
Kolkata – 700 108
www.isical.ac.in/~bhargab

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Subhashis Majumder
International Institute of Information Tech.
Kolkata – 700 091

Susmita Sur-Kolay, Subhas C. Nandy
Bhaskar Chakraborty
Indian Statistical Institute
Kolkata – 700 108

Relevance of the problem

• Dramatic increase in power consumption of present-day ULSI circuits
  – causes very high operating temperature
  – hence affects timing concerns seriously
• By 2010, power density in some regions of chip may rise to 200W/cm² for 45 nm technology
• The need for new thermal designs to cope up with the challenge being thrown by high-scale integration has recently been highlighted [Kang, ISPD, 2002]

Organization of this Talk

• Time-Invariant Thermal Sources
  – Continuous spatial domain
  – Discrete spatial domain
• Time-varying Thermal Sources
• Analysis using Voronoi Diagram
• Concept of Density and Discrepancy
• Conclusion and Future Work

Time-invariant Thermal Sources

• Objective
  – To identify the zones whose heat content crosses a certain threshold
• Assumptions
  – Constantly active (always on) heat-generating sources placed throughout the chip floor
  – No radiation loss: entire heat propagates through the 2D surface of the floor, without dissipation into the ambience

Continuous Spatial Domain

• Position of a heat source may be any point on the chip floor
• Unit-circle model – Contribution of a heat source S at a target point T is the amount of heat received within the unit circle centered at T
• The unit may be related to the layout dimension of the chip and the distance between S and T is being measured by the same unit.
• Cumulative heat at T is the linear superposition of the amounts received from all sources
Unit-circle Model

Contribution of S at T = \( Q \frac{(2\theta d)}{(2\pi d)} = Q \frac{\theta}{\pi} = 2 \frac{Q \sin^2(1/2d)}{\pi} \)

\[
\sin(\theta/2) = \frac{1}{2}/d
\]

2 Special Cases of Unit-circle Model

Case I: \( 0 < d < \frac{1}{2} \)
- \( \theta > \frac{\pi}{3} \)
- Heat received at \( T > Q/3 \)

Case II: \( \frac{1}{2} < d < 1 \)
- \( 0 > \frac{\pi}{3} \)
- Heat received at \( T = Q/6 \)

Illustration of Thermal Aggressor and Victim

Aggressors - heat sources of unit strength

Cumulative heat at
- aggressor : 1.90252
- victim : 1.91534

A Typical Thermal Distribution

10 source points each having unit strength
Threshold = Min cumulative power at any src pt
41 non-src pts crossed the threshold

Discrete Spatial Domain

- Uniform grid of suitable resolution
- Position of a Heat source may be only at some grid point
- We evaluate cumulative power only at grid points
- Simplifies computation of cumulative power
- Accuracy of Propagation model is important

Propagation Model for uniform grid

Power retransmission stops at the boundary
Propagation Model (contd.)

Sum total heat propagating out of each frontier is same and is equal to unity

Class I: \( P(p,0) = \frac{1}{4} 3^{p-1} \), \( P(0,q) = \frac{1}{4} 3^{q-1} \)

Class II: \( P(p,1) = \frac{1}{3} P(p,0) + \frac{1}{2} P(p-1,1) \)

Class III: \( P(p,q) = \frac{1}{2} P(p-1,q) + \frac{1}{2} P(p,q-1) \)

Expression depends on # Catalan paths from R to T

Main Observation in Discrete Domain

- Cumulative heat at few of the passive grid points exceeds that at some of the grid points containing active heat sources

Window Model in the Discrete Domain

- Consider a moving Rectangular Window aligned with the grid
- Window Power – Sum of cumulative heat received at each grid point covered by the window

Observations in the Window Model

- Observation 1 – Some windows that do not contain any active source (see rect. A) record a higher power than some of the windows covering active sources (see rect. B)
- Observation 2 – If several overlapping windows cross threshold, then we should thermally guard the union of those windows

Time-varying Heat Sources

- Most generalized model – active source strength may vary with time
- For some part of the duty cycle the sources may behave as passive points also
- Not yet studied in details by us
- Interesting observation – zones with no active sources not only crosses threshold but often becomes the hottest zone of the floor
Exp. Results - Continuous Domain
- Used Mathematica 5.0 to simulate effect of heat sources placed at random points
- Studied 5 trial runs keeping number and strength of heat sources fixed but positions were varied – results did not vary much
- It was run like a biased Monte Carlo
- Threshold \( P_{th} \) is min cumulative power at any active src pt

<table>
<thead>
<tr>
<th># src pts</th>
<th># probe pts</th>
<th>average # non-src pts crossing ( P_{th} )</th>
<th>average # non-src pts crossing ( P_{th} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4500</td>
<td>68</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>14000</td>
<td>185</td>
<td>1</td>
</tr>
<tr>
<td>20</td>
<td>30500</td>
<td>826</td>
<td>15</td>
</tr>
<tr>
<td>40</td>
<td>56000</td>
<td>1673</td>
<td>35</td>
</tr>
<tr>
<td>50</td>
<td>60000</td>
<td>1871</td>
<td>46</td>
</tr>
</tbody>
</table>

Exp. Results - Discrete Domain
- Implemented in C
- Finite number of sources distributed randomly over a 50x50 grid
- 5 trials varying positions, keeping number and strength of sources fixed
- \( P_{th} \) is similarly defined
- Power was evaluated at all the grid points
- \( n \) is number of non-src pts crossing threshold

<table>
<thead>
<tr>
<th>#probe pts</th>
<th># src pts</th>
<th>average # non-src pts crossing ( P_{th} )</th>
<th>average # non-src pts crossing ( P_{th} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>70230751</td>
<td>48328951</td>
<td>615</td>
<td>615</td>
</tr>
<tr>
<td>48328951</td>
<td>1644651</td>
<td>78</td>
<td>78</td>
</tr>
<tr>
<td>1644651</td>
<td>157751</td>
<td>160</td>
<td>160</td>
</tr>
<tr>
<td>157751</td>
<td>7351</td>
<td>170</td>
<td>170</td>
</tr>
</tbody>
</table>

Exp. Results - Window Model
- 50x50 grid
- Window-threshold \( P_{window} \) – min. cumulative source power \( (P_{th}) \) times number of grid points covered by window
- 4 trials in each case
  - 2 with 3x3 window and 2 with 5x5 window
  - \( n_1 \) – # window positions crossing \( P_{window} \)
  - \( n_2 \) – # window positions covering no active source still crossing \( P_{window} \)

<table>
<thead>
<tr>
<th>Window size</th>
<th>Trial #</th>
<th>( n_1 )</th>
<th>( n_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3x3</td>
<td>5</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>5x5</td>
<td>5</td>
<td>300</td>
<td>400</td>
</tr>
<tr>
<td>3x3</td>
<td>10</td>
<td>1</td>
<td>26</td>
</tr>
<tr>
<td>5x5</td>
<td>10</td>
<td>3</td>
<td>101</td>
</tr>
<tr>
<td>3x3</td>
<td>20</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>5x5</td>
<td>20</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3x3</td>
<td>40</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>5x5</td>
<td>40</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Power Window results with Varying Threshold
- We vary \( P_{th} \) from min cumulative source power to max initial source power
- in turn varies \( P_{window} \)
- We consider last trial from the previous table
  - \# src pts = 40
  - \# points covered by window is 6x6 = 36
  - Observation – \( n_1 \) and \( n_2 \) ↓ as \( P_{window} \) ↑

<table>
<thead>
<tr>
<th>P th</th>
<th>Pwindow</th>
<th>( n_1 )</th>
<th>( n_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.77</td>
<td>243.75</td>
<td>854</td>
<td>243</td>
</tr>
<tr>
<td>9.00</td>
<td>324</td>
<td>299</td>
<td>37</td>
</tr>
<tr>
<td>11.23</td>
<td>404.28</td>
<td>46</td>
<td>0</td>
</tr>
<tr>
<td>13.46</td>
<td>484.56</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15.69</td>
<td>564.84</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>17.92</td>
<td>645.12</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Analysis using Voronoi Diagram
- Use a simple geometric technique to make a fast prediction of the hot spots or zones on a chip
- Required because a full-scale simulation approach is time-consuming
- Simulation is however used in conjunction with the above prediction to make the final prediction more accurate – result is a reduction in the overall simulation overload
- Once identified, some of the hot spots may be dispersed, again following some geometric technique to cool down the chip

Problem Addressed
- Identify the hot zones of a chip containing a number of time-invariant thermal sources
- These sources may be abstracted as point sources with an attached +ve real weight reflecting their strength
Important Observation

- Two factors mainly contribute for making a non-source point to get substantially hot
  - Either this point is located near a strong source
  - Or it is in the vicinity of a number of sources whose cumulative effect is significant

Voronoi Diagram (VD)

- Traditionally known as the solution for Post-office problem
- If there are some sites on a plane providing service, the question is
  - which site is preferable for a person located at \( x \) to get service from?
- VDs tessellate plane into regions around each site to show the region of influence
- Can be generalized in many ways
  - Higher dimensions
  - Different distance metric
  - Weighted – Multiplicative or Additive

An ordinary Voronoi Diagram

Multiplicatively Weighted Voronoi Diagram (MWVD)

- Let there be \( k \) sites on a plane where \( p(i) \) is the \( i \) th site with weight \( w(i) \)
- Distance function
  \[
  d(p, p(i)) = \frac{\text{dis}(p, p(i))}{w(i)}
  \]
  where \( \text{dis} \) is the Euclidean distance between a typical point \( p \) and \( i \)th site \( p(i) \)
- Edges are normally circular
- If 2 sites \( p(1) \) and \( p(2) \) have wt. \( w \) and \( 2w \) respectively, the locus of the point \( p \) separating the 2 sites will satisfy this property
  - If the dist. of \( p \) from \( p(1) \) is \( d \), form \( p(2) \) it will be \( 2d \)

A Typical Example

<table>
<thead>
<tr>
<th>Vor. Cell</th>
<th>Raw Heat</th>
<th>Cum. Heat</th>
<th>% area</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.00</td>
<td>1.86</td>
<td>14.43</td>
</tr>
<tr>
<td>B</td>
<td>2.10</td>
<td>2.89</td>
<td>2.51</td>
</tr>
<tr>
<td>C</td>
<td>1.00</td>
<td>2.28</td>
<td>13.43</td>
</tr>
<tr>
<td>D</td>
<td>1.25</td>
<td>2.56</td>
<td>15</td>
</tr>
<tr>
<td>E</td>
<td>1.00</td>
<td>2.18</td>
<td>9.81</td>
</tr>
<tr>
<td>F</td>
<td>1.50</td>
<td>2.92</td>
<td>67</td>
</tr>
<tr>
<td>G</td>
<td>1.00</td>
<td>2.53</td>
<td>39.29</td>
</tr>
<tr>
<td>H</td>
<td>2.00</td>
<td>3.05</td>
<td>18.2</td>
</tr>
<tr>
<td>I</td>
<td>2.00</td>
<td>3.02</td>
<td>9.98</td>
</tr>
<tr>
<td>J</td>
<td>1.00</td>
<td>1.56</td>
<td>12.32</td>
</tr>
<tr>
<td>K</td>
<td>1.50</td>
<td>2.05</td>
<td>10.58</td>
</tr>
</tbody>
</table>
An Empirical Observation

<table>
<thead>
<tr>
<th>Cum. Heat</th>
<th>Vor. Cell</th>
<th>Vor. Cell</th>
<th>% area</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.05</td>
<td>H</td>
<td>H</td>
<td>1.82</td>
</tr>
<tr>
<td>3.02</td>
<td>I</td>
<td>B</td>
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</tr>
<tr>
<td>2.89</td>
<td>B</td>
<td>D</td>
<td>3.15</td>
</tr>
<tr>
<td>2.58</td>
<td>F</td>
<td>I</td>
<td>3.98</td>
</tr>
<tr>
<td>2.56</td>
<td>D</td>
<td>F</td>
<td>4.67</td>
</tr>
<tr>
<td>2.31</td>
<td>G</td>
<td>E</td>
<td>5.49</td>
</tr>
<tr>
<td>2.18</td>
<td>E</td>
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<td>6.81</td>
</tr>
<tr>
<td>2.08</td>
<td>K</td>
<td>C</td>
<td>7.43</td>
</tr>
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<td>1.86</td>
<td>A</td>
<td>A</td>
<td>8.43</td>
</tr>
<tr>
<td>1.56</td>
<td>J</td>
<td>J</td>
<td>12.32</td>
</tr>
</tbody>
</table>

VD for Unweighted Sources

Superposed Simulation Result for Unweighted sources

Hybrid Strategy

- Using the inverse of raw strengths draw the MWVD (ordinary VD for uniform wt.)
- Routine should also report the area of the cells
- Sort the areas in the ascending order
- Do exhaustive simulation at all the grid points of the cells in the top of the order
- Observation – If we cover 10 to 15% of the total area it is sufficient
- Drawing VD and MWVD are low complexity algorithms – overall saving is significant

Dispersing the Hot Spots

- Assume that thermal violations have occurred even in the presence of thermal vias or heat sinks
- Reducing strength of any heat source requires change at the circuit level
- So, dilate the clusters crossing the threshold
- But, do not disturb the adjacency (neighborhood) relationships
Density or Discrepancy?  
A VLSI Designer’s Dilemma in Hot Spot Analysis

Sources of Uniform Strength

- If all the thermal sources are of uniform strength, points will become unweighted.
- Density of a region $R$ with area $A(R)$ will become

\[
\frac{\text{Density of } R}{A(R)} = \frac{3}{8}
\]

Normalized Density

- Normalized density for a set of unweighted points in an axis-parallel region $R$ on a rectangular floor $F$ with $n$ points is

\[
\Delta(R) = \frac{\#(R)}{n} / \frac{A(R)}{A(F)}
\]

- For the previous example $\Delta(R) = (3/8) / (10/80) = 3 / 100$
- Also $\Delta(F) = 1$, hence normalized
Region of Maximum Density

- **Problem P1** - Find the cluster of \( k \geq 2 \) points in \( S \), such that the minimum area rectangle covering them attains the highest density on the floor \( R \)
  - Note that all earlier results either considers fixed # of points covered by the minimum size rectangle or uses a fixed-size rectangle covering max number of points
  - However in this work both are variable
  - Also we tackle weighted cases
  - \( k = 1 \) makes the case degenerate

Main Result on Maximum Density

- Let each point \( p_i \in S \) have a real +ve weight \( w_i \) and also let \( S' \subset S \) be any cluster of \( k \geq 2 \) points such that no two of them lie on the same horizontal (vertical) line.
- Then \( \exists \) a pair of points \( p_i, p_j \in S' \) such that the density of the smallest rectangle containing \( (p_i, p_j) \) is greater than the density of the smallest rectangle containing \( S' \).

Outline of the Proof

- Consider \( k - 1 \) strips \( R_i \) to \( R_{k-1} \), within the rect. \( R' \) that covers the cluster \( S' \) and then show that there must exist a strip covering only two points (shaded portion) having more density than \( R' \).

Implications

- Max. Density always occur for a cluster of 2 pts
- Search space greatly reduces
- even the naïve algo. to find max. density region will be \( O(n^2) \) – consider each pair at a time
- The unweighted case reduces to finding the smallest axis-parallel rectangle covering exactly two points from \( S \).

Algorithm for the Unweighted Case

- \( O(n^2) \) naïve algorithm works for both weighted and unweighted case
- However for unweighted cases, the concept of monotone matrix can be used
- The problem reduces to finding out minimum area empty corner rectangles
- Complexity of improved algo. - \( O(n \log^2 n) \)

Density in General Case

- In isothetic case, thin rectangles pose problems – reports unnecessarily high area, so consider general case
- In non-isothetic domain also, we get similar results as earlier
- Definition – Let \( S' \subset S \) be a subset of points. The density of \( S' \) is defined as 
  \[ \rho(S') = \frac{|S'|}{A(\text{conv}(S'))} \]
  - where \( |S'| \) is cardinality of \( S' \)
  - \( A(\text{conv}(S')) \) is the area of the convex hull of \( S' \)
**Fact**

Let there be a set $S$ of $n > 2$ points on a rectangular floor $R$ such that no three of those points are co-linear. There exists a cluster $C \subset S$ of three points such that the triangle having those three points as its vertices and covering no other point from the set $S$ achieves the highest density on the floor.

**Outline of the Proof**

- Consider some triangulation of $S$ and then apply the fact that the average of a set of quantities lies within the minimum and the maximum value.

![Diagram showing triangulation with minimum number of triangles](image)

**Discussion on Minimum Density**

- Problem P2 - Find the cluster of $k$ ($k \geq 1$) points from $S$, such that the maximum area rectangle covering them attains the lowest density on the bounded floor $R$.

**Main Result**

- Let each point $p_i \in S$ have a weight $w_i > 0$, and let $S' \subset S$ be a cluster of $k \geq 1$ point(s).
- Then $\exists$ a point $p_i \in S'$ such that the density of the largest rectangle that contains only point $p_i$ is less than the density of the largest rectangle $R'$ containing $S'$ and no other points from $S$.
- Implication – Reduction of search space.

**Outline of the proof**

- Consider $k + 1$ strips $R_0$ to $R_k$ within the rect. $R'$ that covers the cluster $S'$ and then show that there must exist a joint strip covering only one point (shaded portion) having less density than $R'$.

**Discrepancy Measure**

- Studies concentration or rarefaction of a subset $S' \subset S$ of points w.r.t. the orig. set $S$.
- Definition - Assuming that the area containing a point set $S$ is a unit square, the discrepancy measure of $S'$ may be defined as $\Delta = \max \{|\text{area}(r) - (\#r / n)|\}$, where $r \in R'$ and $\# r$ indicates the number of points of $S'$ lying inside $r$.
Local Discrepancy

For a rectangular area \( r \), local discrepancy \( \Delta_l(r) = |\text{area}(r) - (\# r / n)| \) (defined by us)

\[
\Delta_l(r) = |(0.5 * 0.25) - (3/10)| = 0.175
\]

Density and Discrepancy – Compared

• Two set of points having the same density may have their local discrepancy different and vice-versa
• The rise or fall of density function does not map monotonically to the rise or fall of discrepancy function
• The above observation is the main deterrent in interpreting the results of one domain from the other

Conclusions and Future Work

• Proposed models both in the continuous and discrete domain to analyze the thermal behavior of a chip
• Simulation-based results support that zones in the chip may become substantially hot even without containing any active heat source
• Hence identifying and guarding these zones is crucial
• Development of a geometry-based method to predict these hot zones is in progress

Thank you